

**GEOMETRIC AND PHYSICAL INTERPRETATION
OF FRACTIONAL INTEGRATION AND
FRACTIONAL DIFFERENTIATION**

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*Dedicated to Professor Francesco Mainardi,
on the occasion of his 60-th birthday*

Abstract

A solution to the more than 300-years old problem of geometric and physical interpretation of fractional integration and differentiation (i.e., integration and differentiation of an arbitrary real order) is suggested for the Riemann-Liouville fractional integration and differentiation, the Caputo fractional differentiation, the Riesz potential, and the Feller potential. It is also generalized for giving a new geometric and physical interpretation of more general convolution integrals of the Volterra type. Besides this, a new physical interpretation is suggested for the Stieltjes integral.

Mathematics Subject Classification: 26A33, 26A42, 83C99, 44A35, 45D05

Key Words and Phrases: fractional derivative, fractional integral, fractional calculus, geometric interpretation, physical interpretation

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